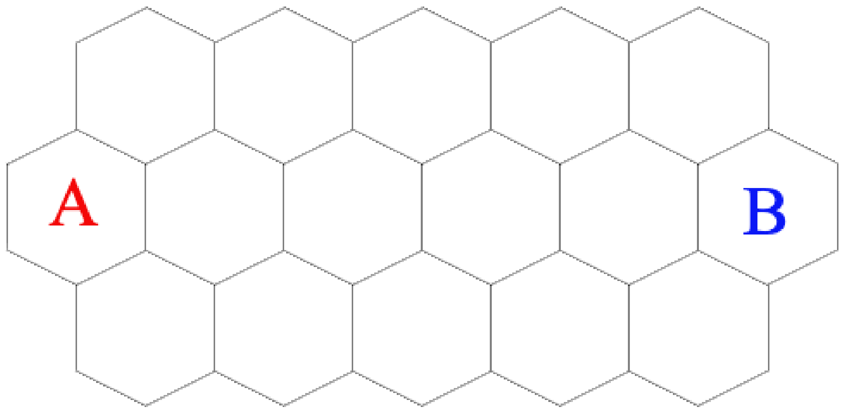
1. Matrix Tessellation Laplace writes the decimal number 2024 in base 3 on a blackboard. He writes another decimal number $0 \leq N \leq 2023$ in base 3 underneath and subtracts the bottom number from the top number. He notices that as he performs the subtraction from right to left, at no point does he need to borrow (regroup). Compute the number of possible values of N.

2. How many ways are there to color each hexagon below either red (denoted by A) or blue (denoted by B) such that the region of all red hexagons is contiguous, the region of all blue hexagons is contiguous, and each hexagon borders at least two hexagons of the same color? The leftmost and rightmost hexagons are already colored.



3. Let O be the center of a unit circle and d be a diameter of this circle. Consider a point $X \neq O$ with $XO \perp d$. Let points $X_1, X_2, \dots$ be the points on line XO such that $\overline{X_iO} = \frac{1}{2^i}$. Let p_i be the chord passing through X_i with $p_i \perp XO$ and let C_i and B_i be the endpoints of p_i. If A_i is the area of the triangle OB_iC_i, compute

$$\sum_{i=1}^{\infty} A_i^2.$$

4. Triangle ABC has $AB = 13, BC = 14, CA = 15$. A circle with the same radius as the incircle of $\triangle ABC$ is centered at vertex A of $\triangle ABC$. It is then slid along side AB to become centered at point B, then slid along slid BC to become centered at point C, and finally slid along CA to become centered at point A. Find the perimeter of the region swept out by the circle.

5. There are 4 outlets A, B, C, D in a room, each with 2 sockets denoted with subscripts 1 and 2 (i.e. outlet A has sockets A_1 and A_2). A baby starts plugging wires into the sockets. They have 3 wires, and each wire has two ends that each plug into a socket. Each socket can take at most one plug.

   A short circuit happens when there is a loop from one outlet to itself. For example, $A_1 \leftrightarrow A_2$ is a short circuit and $A_1 \leftrightarrow B_1, B_2 \leftrightarrow C_1, A_2 \leftrightarrow C_2$ is a short circuit, but $A_1 \leftrightarrow B_1$ is not.

   Provided that the baby randomly connects every plug to a socket, what is the probability that the baby causes a short circuit?

6. Point P lies inside rectangle $ABCD$ so that $PA = 52, PB = 60$, and $PD = 25$. Let $\overrightarrow{AP}$ intersect CD at A', $\overrightarrow{BP}$ intersect CD at B', and $\overrightarrow{CP}$ intersect AD at C'. If $AB = 56$, what is the ratio of the area of $\triangle PA'B'$ to the area of $\triangle PC'D$?

7. There are two birds currently sitting on a tree. At the end of every passing hour, there is a $1 - \frac{1}{n}$ probability that one of the birds on the tree at that time leaves the tree (where n is the number of birds sitting on the tree that past hour). Similarly, at the end of every passing hour there is an independent probability of $\frac{1}{n}$ that one new bird will come sit on the tree. What is the expected number of hours that will pass until there are four birds on the tree for the first time?

8. Find the number of ordered pairs of positive integers (a, b) such that $a^2 + b^2$ is a divisor of 2024^2.

9. Dean's life is going in circles. Help him escape! Dean is standing in the center of a unit circle. At each moment he can only go east or north-east (at a 45° angle). He walks until he reaches the circumference of the circle. Compute the square of the length of the longest path he could take.

10. A group of scientists are researching the population dynamics of a certain bacterial colony. They find that if P_n represents the bacterial population at n minutes from the start, then this population can be modeled in a surprisingly discrete way given by $P_n = \frac{P_{n-1}^3}{P_{n-3}^4}$. Given that in one experiment, $P_0 = 1, P_1 = 1$, and $P_2 = 7^3$, find $3\log_7(P_{100})$.

11. Floria is picking flowers on a vast prairie with six different colors of flowers, where each color is equally likely to be picked at random, independent of previously picked flowers. Out of the flowers she picks, Floria wants to make a bouquet of 7 flowers, such that all 6 colors are used and there is a pair of identically colored flowers. If she picks flowers, one by one, at random, what is the expected number of flowers Floria needs to pick in order to make the bouquet?

12. Suppose ABC is a triangle with $AB = 13$, $BC = 15$, and $AC = 14$. Let H be the orthocenter of $\triangle ABC$, and let M be the midpoint of BC. If P is the intersection of the circumcircle of $\triangle ABC$ with line MH that lies on minor arc AB, compute the length of HP.

13. Two parabolas $y = (x-1)^2 + a$ and $x = (y-1)^2 + b$ intersect at a single point, where a and b are non-negative real numbers. Let c and d denote the minimum and maximum possible values of $a + b$, respectively. Compute $\lfloor c \rfloor + d$.

14. Let $f(n)$ be the number of positive divisors of n that are of the form $4k + 1$, for some integer k. Find the number of positive divisors of the sum of $f(k)$ across all positive divisors of $2^8 \cdot 29^{59} \cdot 59^{79} \cdot 79^{29}$.

15. Triangle ABC has side lengths $AB = 24$ and $BC = 23$, and is inscribed in the circle ω. The radius of ω is 15 and point P lies on minor arc BC of ω. Let M be the midpoint of AB. Line PM intersects ω at $F \neq P$. Let T be the intersection of the tangents to ω passing through A and B. Let TF intersect AB at K, and ω at $L \neq F$. Finally, let FC intersect BP at Q, and let TQ intersect BC at N. If $TL = 25$, compute the area of $\triangle KMN$.