1. Let $ABCD$ be a square, and let P be a point chosen on segment AC. There is a point X on segment BC such that $PX = PB = 37$ and $BX = 24$. Compute the side length of $ABCD$.

2. Let $\triangle ABC$ be an equilateral triangle with side length 6. Three circles of radius 6 are centered at A, B, and C. Compute the radius of the circle that is centered at the center of $\triangle ABC$, is internally tangent to these three circles, and lies in the interior of the three circles.

3. Let ω be the circle inscribed in regular hexagon $ABCDEF$ with side length 1, and let the midpoint of side BC be M. Segment AM intersects ω at point $P \neq M$. Compute the length of AP.

4. Let $\triangle OAB$ and $\triangle OA'B'$ be equilateral triangles such that $\angle AOA' = 90^\circ$, $\angle BOB' = 90^\circ$, and $\angle AOB'$ is obtuse. Given that the side length of $\triangle OA'B'$ is 1 and the circumradius of $\triangle OAB'$ is $\sqrt{61}$, compute the side length of $\triangle OAB$.

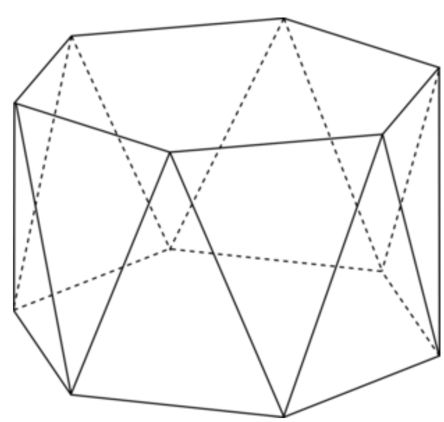
5. In right triangle $\triangle ABC$ with right angle at B, let I be the incenter and G the centroid. Let the foot of the perpendicular from I to AB be D and the foot of the perpendicular from G to CB be E. Line l is drawn such that l is parallel to DE and passes through B. Line ID meets l at X, and line GE meets l at Y. Given that $AB = 8$ and $CB = 15$, compute the length XY.

6. Let $ABCDE$ be a regular pentagon with side length 1. Circles ω_B, ω_C, and ω_D are centered at B, C, and D respectively, each with radius 1. ω_B intersects ω_C inside $ABCDE$ at point F, and ω_C intersects ω_D inside $ABCDE$ at point G. Compute the ratio of the measure of $\angle AFB$ to the measure of $\angle AGB$.

7. Consider the horizontal line that intersects the ellipse $\frac{x^2}{9} + y^2 = 1$ at points A and B above the x-axis such that $\angle AOB = 120^\circ$, where point O is the origin. Compute the area of the region of the ellipse that lies above this line.

8. Points A and B lie on a circle centered at O such that $AB = 14$. The perpendicular bisector of AB intersects $\odot O$ at point C such that O lies in the interior of $\triangle ABC$ and $AC = 35\sqrt{2}$. Lines BO and AC intersect at point D. Compute the ratio of the area of $\triangle DOC$ to the area of $\triangle DBC$.

9. Consider a prism with regular hexagon bases. We form an antiprism by removing the lateral faces, rotating one of the bases 30° about the axis passing through the centers of the bases, and forming 12 triangular faces between the bases where each triangular face consists of one vertex of one of the bases and the two closest vertices of the other base. Compute the ratio of the volume of the antiprism to the volume of the original prism.



10. Given a triangle ABC, let the tangent lines to the circumcircle of $\triangle ABC$ at points A and B intersect at point T. Line CT intersects the circumcircle for a second time at point D. Let the projections of D onto AB, BC, AC be M, N, P respectively. From M draw a line perpendicular to NP intersecting BC at E. If $EC = 5$, $EB = 11$, and $\angle CEP = 120^\circ$, compute the length of CP.