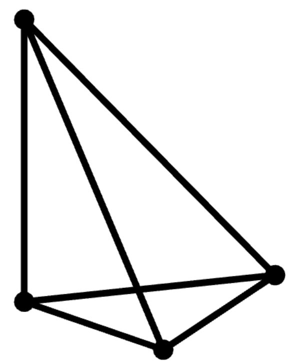
1. Find the volume of the pyramid with vertices at the coordinates

$$(1,0,0),(0,1,0),(0,0,1),(0,0,0).$$



2. Consider the following system of equations:

$$w+x+y=8$$

$$y+z=10$$

$$w+x+z=12.$$

Find $w+x+y+z$.

3. What is the number of 5-digit numbers that have strictly decreasing digits from left to right?

4. Call a number balanced if it is divisible by $p+10$ where p is its smallest prime divisor. How many numbers from 1 to 100, inclusive, are balanced?

5. Let $\mathcal{A}$ be the region in the xy-plane bounded by $y=0$, $y=x$, and $y=2-x$. $\mathcal{A}$ includes the area enclosed by these boundaries, as well as the boundaries themselves. What is the maximum possible radius of a circle that lies in $\mathcal{A}$?

6. Find the area enclosed by the relation:

$$|x+y|+|x-y|=16.$$

7. What is the probability of obtaining a sum of 9 by rolling 4 six-sided dice?

8. For some real constant c, the roots of the quadratic $x^2+cx-2024$ are r and s. If the quadratic x^2+rx+s has one distinct root t (not necessarily real), find t.

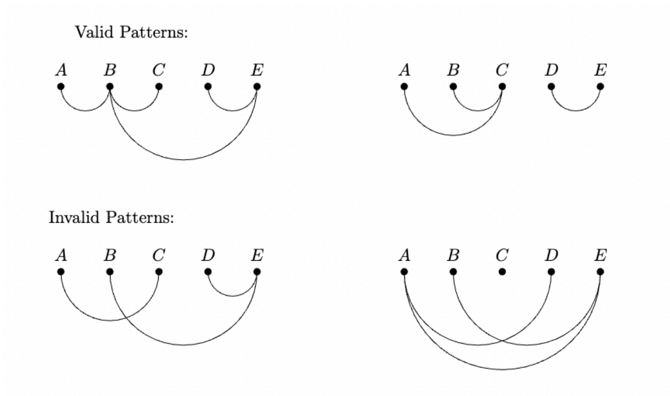
9. Call two positive integers *similar* if their prime factorization have the same number of distinct prime divisors, and when ordered in some way, the exponents match. For example, 250 and 24 are *similar* because $250=5^3\cdot 2$, and $24=2^3\cdot 3$. How many positive integers less than or equal to 200 are *similar* to 18 (including itself)?

10. What is the sum of the possible values of c such that the polynomial $x^2-40x+c=0$ has positive integer roots (possibly equal to each other)?

11. Let $\triangle ABC$ be an equilateral triangle with side length 6. Three circles of radius 6 are centered at A, B, and C. Compute the radius of the circle that is centered at the center of $\triangle ABC$, is internally tangent to these three circles, and lies in the interior of the three circles.

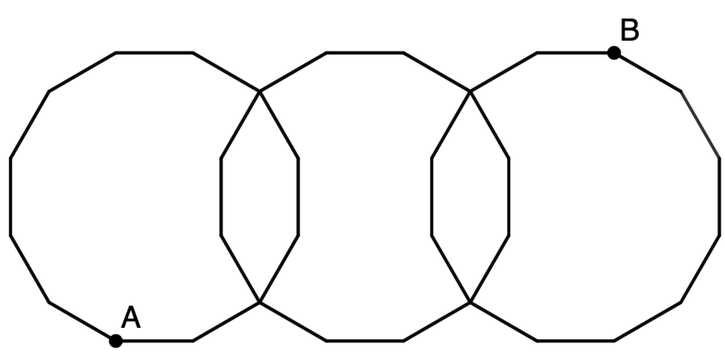
12. Compute the largest positive integer x less than 1000 that satisfies $x^2 \equiv 24 \pmod{1000}$.

13. Jana is decorating her room by hanging zero or more strings of lights. She has 5 collinear attachment points (A, B, C, D, and E), and she can connect any two attachment points with a semicircular string of lights (direction hanging downward), as long as no two strings cross. In how many different patterns can she hang the lights?



14. Let $0.\overline{d}_b = 0.ddd..._b$ denote a repeating decimal written in base b. If $0.\overline{4}_a + 0.\overline{7}_b = 1$ for positive integers a and b such that $a \neq b$, what is the minimum possible value of $a + b$?

15. Perry bakes a pineapple upside-down cake with 3 slices of pineapple on top, each in the shape of regular dodecagons of side length 1. The pineapple slices overlap each other as shown in the figure. Compute the length of the cut AB that Perry makes to slice the cake in half.



16. Let $P(n)$ represent the number of real roots x for the equation

$$x^n + x^{n-1} + ... + x^1 + 1 = 0.$$

Compute

$$P(1) + P(2) + P(3) + ... + P(2024).$$

17. Nacho is building a sandcastle. Each time he adds a scoop of sand, he has a $\frac{5}{6}$ chance that the sandcastle will increase by 1 inch in height. Nacho is a clumsy engineer, so each time the height doesn't increase, the sandcastle topples and loses $\frac{1}{3}$ of its current height. Suppose Nacho starts his sandcastle at height H. What H should he choose so that after any number of scoops, the expected height of his sandcastle is still H?

18. Consider the horizontal line that intersects the ellipse $\frac{x^2}{9} + y^2 = 1$ at points A and B above the x-axis such that $\angle AOB = 120^\circ$, where point O is the origin. Compute the area of the region of the ellipse that lies above this line.

19. Let $a_1, a_2, \ldots$ be a strictly increasing sequence of positive integers such that a_{3k-2} is divisible by 8 and a_{3k} is divisible by 9 for all positive integers k. Find the largest possible positive integer i such that $a_i > 2024$ and $a_{i-1} \leq 2024$.

20. I have 3 red balls, 3 blue balls, and 3 yellow balls. These 9 balls are randomly arranged on a 3×3 grid. Let a *3-in-a-row* denote when 3 balls of the same color are aligned in a line in the grid (a row, column, or diagonal consisting of 3 balls of the same color). What is the expected number of *3-in-a-rows* that will show up in the grid?

21. How many positive integers n are there such that the powers of $2024 \pmod n$ repeat in a cycle of length 2? In other words, how many positive integers n are there such that

$$2024^0 \pmod n \equiv 2024^2 \pmod n \equiv 2024^4 \pmod n \ldots$$

and

$$2024^1 \pmod n \equiv 2024^3 \pmod n \equiv 2024^5 \pmod n \ldots$$

but

$$2024^0 \pmod n \not\equiv 2024^1 \pmod n?$$

22. Consider a quadratic function $P(x) = ax^2 + bx + c$ with distinct positive roots r_1 and r_2, and a second polynomial $Q(x) = cx^2 + bx + a$ with roots r_3 and r_4. John writes four numbers on the whiteboard: r_1, r_2, $4r_3$ and $4r_4$. What is the smallest possible integer value of the sum of the numbers John wrote down?

23. Let $ABCDE$ be a regular pentagon with side length 1. Circles ω_B, ω_C, and ω_D are centered at B, C, and D respectively, each with radius 1. ω_B intersects ω_C inside $ABCDE$ at point F, and ω_C intersects ω_D inside $ABCDE$ at point G. Compute the ratio of the measure of $\angle AFB$ to the measure of $\angle AGB$.

24. Let F be a set of subsets of $\{1, 2, 3\}$. F is called *distinguishing* if each of 1, 2, and 3 are distinguishable from each other—that is, 1, 2, and 3 are each in a distinct set of subsets from each other. For example $F = \{\{1, 3\}, \{2, 3\}\}$ is *distinguishing* because 1 is in $\{1, 3\}$, 2 is in $\{2, 3\}$, and 3 is in $\{1, 3\}$ and $\{2, 3\}$. $F = \{\{1, 2\}, \{2\}\}$ is also *distinguishing*: 1 is in $\{1, 2\}$, 2 is in $\{1, 2\}$ and $\{2\}$, and 3 is in none of the subsets.

On the other hand, $F = \{\{1\}, \{2, 3\}\}$ is not *distinguishing*. Both 2 and 3 are only in $\{2, 3\}$, so they cannot be distinguished from each other.

How many *distinguishing* sets of subsets of $\{1, 2, 3\}$ are there?

25. For the numbers $4^4, 4^{44}, 4^{444}, \ldots, 4^{44\ldots4}$ where the last exponent is 2024 digits long, Quincy writes down the remainders when they are divided by 2024. Compute the sum of the distinct numbers that Quincy writes.