1. Kevin evaluates the sum of all positive divisors of 10000 that are multiples of 100 and writes the result on a blackboard. Underneath, he evaluates the sum of all positive divisors of 100 and writes down that result. Compute the ratio of the top number to the bottom number.

2. Let $X$ be a 2024 digit perfect square. Let $a(X)$ be the 1012 digit number formed from reading the first 1012 digits of $X$, in order, and let $b(X)$ be the 1012 digit number formed from reading the last 1012 digits of $X$, in order. Given that $X$ is the unique choice that maximizes $a(X) - b(X)$, find the sum of digits of $X$.

3. Let $F$ be a set of subsets of $\{1, 2, 3\}$. $F$ is called *distinguishing* if each of 1, 2, and 3 are distinguishable from each other—that is, 1, 2, and 3 are each in a distinct set of subsets from each other. For example $F = \{\{1, 3\}, \{2, 3\}\}$ is *distinguishing* because 1 is in $\{1, 3\}$, 2 is in $\{2, 3\}$, and 3 is in $\{1, 3\}$ and $\{2, 3\}$. $F = \{\{1, 2\}, \{2\}\}$ is also *distinguishing*: 1 is in $\{1, 2\}$, 2 is in $\{1, 2\}$ and $\{2\}$, and 3 is in none of the subsets.

   On the other hand, $F = \{\{1\}, \{2, 3\}\}$ is not *distinguishing*. Both 2 and 3 are only in $\{2, 3\}$, so they cannot be distinguished from each other.

   How many *distinguishing* sets of subsets of $\{1, 2, 3\}$ are there?

4. Each vertex and edge of an equilateral triangle is randomly labelled with a distinct integer from 1 to 10, inclusive. Compute the probability that the number on each edge is the sum of those on its vertices.

5. We define the *spillage* of a number as $s(x) = \left\lfloor \frac{x}{100} \right\rfloor$, that is, the largest integer that is at most $\frac{x}{100}$. The *spillage* of a list of numbers $[a_1, a_2, \ldots, a_n]$ is the sum of left to right *spillages*: $s(a_1) + s(a_1 a_2) + s(a_1 a_2 a_3) + \ldots + s(a_1 a_2 \cdots a_n)$. Let $M$ be the minimum possible *spillage* of $[1, 2, \ldots, 10]$ over all the permutations of this list. How many of these permutations achieve $M$?

6. Alice is playing with magnets on her fridge. She has 7 magnets, with the numbers $1, 2, 3, 4, 5, 6, 7$, in that order in a row, and she also has two magnets with a "$+$" sign, two magnets with a "$-$" sign, and two magnets with a "$\times$" sign. She randomly puts these six operation magnets between her 7 number magnets, with one operation between every two consecutive numbers, and evaluates the resulting expression (following the order of operations). What is the expected value of her result?

7. Let $1 \leq A \leq 119$ and $1 \leq B \leq 139$ be two integers such that $\frac{A}{60}$ and $\frac{B}{70}$ are fractions in simplest form, yet, when adding $\frac{A}{60}$ and $\frac{B}{70}$ by rewriting both fractions with their lowest common denominator and adding the resulting numerators, the new fraction can be simplified. Find how many ordered pairs $(A, B)$ are possible.

8. Call a polynomial $P$ *cool* if it has degree less than 257, each of its coefficients are nonnegative integers less than 257, and

$$\sum_{k=0}^{256} P(k^j)$$

   is divisible by 257 for all positive integers $j$. How many *cool* polynomials are there? (Assume that the polynomial $P(x) = 0$ has degree less than 257.)

9. Let $f : \mathbb{N} \to \mathbb{R}$ be a function which satisfies $\frac{1}{64} n^2 = \sum_{d|n} f(d) f(\frac{n}{d})$. What is the least integer $n$ for which $f(n)$ is an integer?

10. There are $2n$ students taking an exam, and at the beginning they all put their phones into a pile. When leaving, each person takes an arbitrary phone from the pile. Unfortunately, it might be the case that some students did not get back their own phone!

    To get back the correct phones, the students come up with the following strategy. They repeat the following *round* as many times as needed:

    1. Some of the students pair up. Each student can be in at most one pair.
    2. The pairs swap phones according to some swap order (i.e. an ordering of the pairs).

    For a given assignment $\pi$ of the $2n$ students to the phones they originally picked up, let $r(\pi)$ be the minimum number of rounds required for the students to each receive back their own phone, assuming the students make swaps optimally. Let $s(\pi)$ be the number of ways to swap phones (determined by pairings and swap orders over all rounds) achieving $r(\pi)$ rounds. Let $M(n)$ be the maximum value of $r(\pi)$ over all assignments $\pi$, and let $f(n)$ be the sum of $s(\pi)$ over all $\pi$ with $r(\pi) < M(n)$.

    Then, there exists a unique ordered pair $(a, b)$ with $a > 0$ and $b > 0$ such that $\lim_{n\to\infty} \frac{f(n)\cdot a^n}{(2n)!} = b$. Compute $(a, b)$.

    Note: It may be helpful to know that $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$.