1. Compute

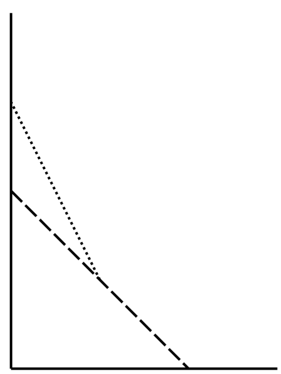
$$\lim_{x\to\infty} \frac{\int_0^x e^{t^2}\,dt}{e^{x^2}}.$$

2. Consider the region specified by the union of the inequalities $1 \geq y \geq x^2$ and $1 \geq x \geq y^2$. What is the volume of the solid created by rotating this region about the x-axis?

3. Compute the limit

$$\lim_{x\to 0} \frac{g'(x)}{x^6}, \text{where } g(x) = \int_0^{x^4} \frac{xte^{\frac{t}{x}}}{x^2+t^2}dt.$$

4. Consider the pair of ladders shown in the below image. The bottom ladder (dashed line) connects the points $(0,4)$ and $(4,0)$. The top ladder (dotted line) is attached to the bottom ladder at $(2,2)$ and touches the wall at $(0,6)$. This pair of ladders begins to slide down the wall, such that the top ladder remains attached to the midpoint of the bottom ladder. When the bottom ladder touches the wall at the point $(0,2)$, the end at which it touches the wall is moving downward at the rate of 2 units per second. At that point in time, what is the rate at which the wall end of the top ladder is moving downward, in units per second?



5. Charlie chooses a real number $c > 0$. Bob chooses a real number b from the inteval $(0, 2c)$ uniformly at random. Alice chooses a real number $\tilde{a}$ from the interval $(0, 2\sqrt{c})$ uniformly at random. Let $a = \tilde{a}^2$. What is the probability that the roots of $ax^2 + bx + c$ have nonzero imaginary parts and have real parts with absolute value greater than 1?

6. Let $f_0(x) = \max(|x|, \cos(x))$, and $f_{k+1}(x) = \max(|x| - f_k(x), f_k(x) - \cos(x))$. Compute

$$\lim_{n\to\infty} \int_{-\pi/2}^{\pi/2} (f_{n-1}(x) + f_n(x))\,dx.$$

7. If $f(x)$ is a non-negative differentiable function defined over positive real numbers that satisfies $f(1) = \frac{25}{16}$ and

$$f'(x) = 2x^{-1}f(x) + x^2\sqrt{f(x)},$$

compute $f(2)$.

8. Compute

$$\int_0^1 \cos\left(\frac{\pi}{4} + \frac{10^4 - 1}{2} \arccos(x)\right) dx.$$

9. The integral $\int_0^{f(2024)} f^{-1}(x)\, dx$, where $f(x) = \int_0^x e^{-t^2}\, dt$, can be written in the form $A(1 - e^{-B})$ for positive rational constants A and B. Compute $A + \lfloor \log_{10} B \rfloor$.

10. Compute

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\left(\frac{1}{4}\right)^{m+n}}{(2n+1)(m+n+1)}.$$