



1. Compute the number of solutions $x \in [0, 2\pi]$ to

$$\cos(x^2) + \sin(x^2) = 0.$$

2. Suppose that for some angle $0 \leq \theta \leq \pi/4$, the roots of $x^2 + ax + \frac{3}{10}$ are $\sin \theta$ and $\cos \theta$. If the roots of $p(x) = x^2 + cx + d$ are $\sin(2\theta)$ and $\cos(2\theta)$, what is the value of $p(1)$?

3. Let $q(x) = x^3 - 9x^2 + 18x + 27$. Compute

$$q(-10) + q(-8) + q(-6) + \dots + q(16).$$

4. Compute

$$\frac{5}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{7}{2 \cdot 3 \cdot 4 \cdot 5} + \dots + \frac{99}{48 \cdot 49 \cdot 50 \cdot 51}.$$

5. Ashley writes the concatenation of $\lfloor 2.5^1 \rfloor, \lfloor 2.5^2 \rfloor, \dots, \lfloor 2.5^{1000} \rfloor$ on the board. Her number is 199667 digits long. Now, Bob writes the concatenation of $4^1, 4^2, \dots, 4^{1000}$ on the board. Compute the number of digits in Bob's number.

6. Let $f(x, y) = xy$ and $g(x, y) = x^2 - y^2$. If a counterclockwise rotation of θ radians about the origin sends $g(x, y) = a$ to $f(x, y) = b$, compute the value of $\frac{a}{b \tan \theta}$.

7. Find the number of lines of symmetry that pass through the origin for

$$|xy(x + y)(x - y)| = 1.$$

8. Let a_1, a_2, a_3 , and a_4 be non-negative real numbers such that $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 1$. Compute the minimum possible value of

$$6a_1^3 + 8a_2^3 + 12a_3^3 + 24a_4^3.$$

9. Compute

$$\sum_{a=0}^{100} \sum_{b=0}^{100} \frac{1}{1 + \cos\left(\frac{2\pi(a-b)}{101}\right)}.$$

10. Call a polynomial $x^8 + b_7x^7 + \dots + b_1x + 1$ *binary* if each b_i is either 0 or 1. Compute the number of binary polynomials that have at least one real root.