

Time limit: 15 minutes.

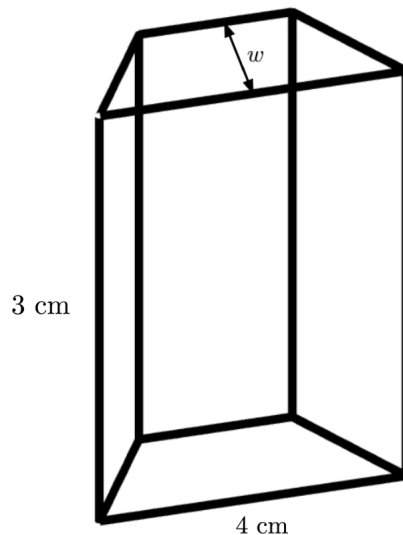
Instructions: This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will not be informed whether your answer is correct until the end of the tiebreaker. You may submit multiple times for any of the problems, but **only the last submission for a given problem will be graded**. The participant who correctly answers the most problems wins the tiebreaker, with ties broken by the time of the last correct submission.

No calculators.

1. Compute

$$\frac{\int_{-\infty}^{\infty} x^{102} e^{1-x^4} dx}{\int_{-\infty}^{\infty} x^{98} e^{1-x^4} dx}.$$

2. A block of cheese in the shape of a triangular prism with height 3 cm and equilateral bases of side length 4 cm is grated along a plane parallel to one of its lateral (non-base) faces. The grater is initially placed at the edge of the prism opposite the lateral face, and over time the prism is truncated along this edge. Call the “width” of the cheese the distance between the grater and the lateral face. The cheese that is grated falls into a funnel that is a square pyramid with height $3\sqrt{3}$ cm and base of side length 4 cm; the cheese falls through the square base at the top toward the vertex. Assume that the grated cheese is fine enough to fill the funnel continuously. If the width of the cheese decreases at a rate of 1 cm/s when the width is $2\sqrt{3} - \sqrt{2}$ cm, the height of the cheese in the funnel increases at a rate of d cm/s. Compute d .



3. A frog hops on the real number line, starting from the origin. Each second, it moves right uniformly at random a distance between 0 and 1. There is an abyss between 1 and $\frac{5}{4}$, and if the frog lands there it will fall into the abyss. If the frog makes it to or passes 3 without falling in the abyss, then the frog is safe. What is the probability it is safe?