Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
No calculators.

1. Points A, B, C, and D lie on a circle. Let AC and BD intersect at point E inside the circle. If $[ABE] \cdot [CDE] = 36$, what is the value of $[ADE] \cdot [BCE]$? (Given a triangle $\triangle ABC$, $[ABC]$ denotes its area.)

2. Let ABC be an acute, scalene triangle. Let H be the orthocenter. Let the circle going through B, H, and C intersect CA again at D. Given that $\angle ABH = 20^\circ$, find, in degrees, $\angle BDC$.

3. $\triangle ABC$ has side lengths 13, 14, and 15. Let the feet of the altitudes from A, B, and C be D, E, and F, respectively. The circumcircle of $\triangle DEF$ intersects AD, BE, and CF at I, J, and K respectively. What is the area of $\triangle IJK$?

4. Let ABC be a triangle with $\angle A = \frac{135}{2}^\circ$ and $\overline{BC} = 15$. Square $WXYZ$ is drawn inside ABC such that W is on AB, X is on AC, Z is on BC, and triangle ZBW is similar to triangle ABC, but WZ is not parallel to AC. Over all possible triangles ABC, find the maximum area of $WXYZ$.

5. In quadrilateral $ABCD$, $AB = 20$, $BC = 15$, $CD = 7$, $DA = 24$, and $AC = 25$. Let the midpoint of AC be M, and let AC and BD intersect at N. Find the length of MN.

6. Let the incircle of $\triangle ABC$ be tangent to AB, BC, AC at points M, N, P, respectively. If $\measuredangle BAC = 30^\circ$, find $\frac{[BPC]}{[ABC]} \cdot \frac{[BMC]}{[ABC]}$, where $[ABC]$ denotes the area of $\triangle ABC$.

7. $\triangle ABC$ has side lengths $AB = 20$, $BC = 15$, and $CA = 7$. Let the altitudes of $\triangle ABC$ be AD, BE, and CF. What is the distance between the orthocenter (intersection of the altitudes) of $\triangle ABC$ and the incenter of $\triangle DEF$?

8. Let Γ and Ω be circles that are internally tangent at a point P such that Γ is contained entirely in Ω. Let A, B be points on Ω such that the lines PB and PA intersect the circle Γ at Y and X respectively, where $X, Y \neq P$. Let O_1 be the circle with diameter AB and O_2 be the circle with diameter XY. Let F be the foot of Y on XP. Let T and M be points on O_1 and O_2 respectively such that TM is a common tangent to O_1 and O_2. Let H be the orthocenter of $\triangle ABP$. Given that $PF = 12, FX = 15, TM = 18, PB = 50$, find the length of AH.

9. The bisector of $\angle BAC$ in $\triangle ABC$ intersects BC in point L. The external bisector of $\angle ACB$ intersects $\overrightarrow{BA}$ in point K. If the length of AK is equal to the perimeter of $\triangle ACL$, $LB = 1$, and $\angle ABC = 36^\circ$, find the length of AC.

10. Let $ABCDEFGH$ be a regular octagon with side length $\sqrt{60}$. Let $\mathcal{K}$ denote the locus of all points K such that the circumcircles (possibly degenerate) of triangles HAK and DCK are tangent. Find the area of the region that $\mathcal{K}$ encloses.