Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
No calculators.

1. If $f(x) = x^4 + 4x^3 + 7x^2 + 6x + 2022$, compute $f'(3)$.

2. The straight line $y = ax + 16$ intersects the graph of $y = x^3$ at 2 distinct points. What is the value of a?

3. For $k = 1, 2, ...$, let f_k be the number of times
$$\sin\left(\frac{k\pi x}{2}\right)$$
attains its maximum value on the interval $x \in [0, 1]$. Compute
$$\lim_{k\to\infty} \frac{f_k}{k}.$$

4. Evaluate the integral:
$$\int_{\frac{\pi^2}{4}}^{4\pi^2} \sin\left(\sqrt{x}\right) dx.$$

5. A net for a hexagonal pyramid is constructed by placing a triangle with side lengths x, x, and y on each side of a regular hexagon with side length y. What is the maximum volume of the pyramid formed by the net if $x + y = 20$?

6. Let
$$f(x) = \cos\left(x^3 - 4x^2 + 5x - 2\right).$$
If we let $f^{(k)}$ denote the kth derivative of f, compute $f^{(10)}(1)$. For the sake of this problem, note that $10! = 3628800$.

7. Let
$$A_j = \left\{(x, y) : 0 \le x \sin\left(\frac{j\pi}{3}\right) + y \cos\left(\frac{j\pi}{3}\right) \le 6 - \left(x \cos\left(\frac{j\pi}{3}\right) - y \sin\left(\frac{j\pi}{3}\right)\right)^2\right\}$$
The area of $\cup_{j=0}^{5} A_j$ can be expressed as $m\sqrt{n}$. What is the area?

8. Given that
$$A = \sum_{n=1}^{\infty} \frac{\sin(n)}{n},$$
determine $\lfloor 100A \rfloor$.

9. Let $f(x, y) = (\cos x + y \sin x)^2$. We may express $\max_x f(x, y)$, the maximum value of $f(x, y)$ over all values of x for a given fixed value of y, as a function of y, call it $g(y)$. Let the smallest positive value x which achieves this maximum value of $f(x, y)$ for a given y be $h(y)$. Compute
$$\int_1^{2+\sqrt{3}} \frac{h(y)}{g(y)}\, \mathrm{d}y.$$

10. Consider the set of continuous functions f, whose n^{th} derivative exists for all positive integer n, satisfying $f(x) = \frac{\mathrm{d}^3}{\mathrm{d}x^3} f(x)$, $f(0) + f'(0) + f''(0) = 0$, and $f(0) = f'(0)$. For each such function f, let $m(f)$ be the smallest nonnegative x satisfying $f(x) = 0$. Compute all possible values of $m(f)$.