**Time limit:** 50 minutes.
**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
**No calculators.**

1. A paper rectangle $ABCD$ has $AB = 8$ and $BC = 6$. After corner $B$ is folded over diagonal $AC$, what is $BD$?

2. Let $ABCD$ be a trapezoid with bases $AB = 50$ and $CD = 125$, and legs $AD = 45$ and $BC = 60$. Find the area of the intersection between the circle centered at $B$ with radius $BD$ and the circle centered at $D$ with radius $BD$. Express your answer as a common fraction in simplest radical form and in terms of $\pi$.

3. If $r$ is a rational number, let $f(r) = (\frac{1-r^2}{1+r^2}, \frac{2r}{1+r^2})$. Then the images of $f$ forms a curve in the $xy$ plane. If $f(1/3) = p_1$ and $f(2) = p_2$, what is the distance along the curve between $p_1$ and $p_2$?

4. $\triangle A_0B_0C_0$ has side lengths $A_0B_0 = 13$, $B_0C_0 = 14$, and $C_0A_0 = 15$. $\triangle A_1B_1C_1$ is inscribed in the incircle of $\triangle A_0B_0C_0$ such that it is similar to the first triangle. Beginning with $\triangle A_1B_1C_1$, the same steps are repeated to construct $\triangle A_2B_2C_2$, and so on infinitely many times. What is the value of $\sum_{i=0}^{\infty} A_iB_i$?

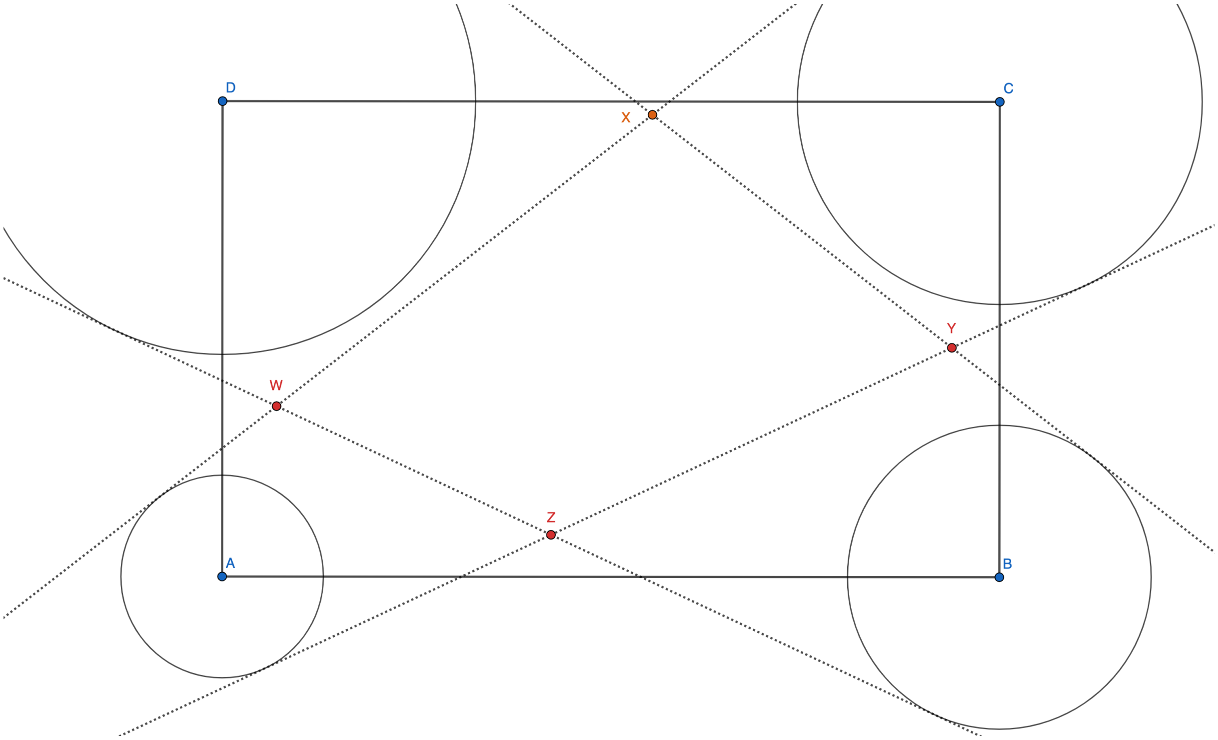
5. Let $ABCD$ be a square of side length 1, and let $E$ and $F$ be on the lines $AB$ and $AD$, respectively, so that $B$ lies between $A$ and $E$, and $D$ lies between $A$ and $F$. Suppose that $\angle BCE = 20^\circ$ and $\angle DCF = 25^\circ$. Find the area of triangle $\Delta EAF$.

6. $\odot A$, centered at point $A$, has radius 14 and $\odot B$, centered at point $B$, has radius 15. $AB = 13$. The circles intersect at points $C$ and $D$. Let $E$ be a point on $\odot A$, and $F$ be the point where line $EC$ intersects $\odot B$ again. Let the midpoints of $DE$ and $DF$ be $M$ and $N$, respectively. Lines $AM$ and $BN$ intersect at point $G$. If point $E$ is allowed to move freely on $\odot A$, what is the radius of the locus of $G$?

7. An $n$-sided regular polygon with side length 1 is rotated by $\frac{180}{n}^\circ$ about its center. The intersection points of the original polygon and the rotated polygon are the vertices of a $2n$-sided regular polygon with side length $\frac{1-\tan^2 10^\circ}{2}$. What is the value of $n$?

8. In triangle $\triangle ABC$, $AB = 5$, $BC = 7$, and $CA = 8$. Let $E$ and $F$ be the feet of the altitudes from $B$ and $C$, respectively, and let $M$ be the midpoint of $BC$. The area of triangle $MEF$ can be expressed as $\frac{a\sqrt{b}}{c}$ for positive integers $a$, $b$, and $c$ such that the greatest common divisor of $a$ and $c$ is 1 and $b$ is not divisible by the square of any prime. Compute $a + b + c$.

9. Rectangle $ABCD$ has an area of 30. Four circles of radius $r_1 = 2, r_2 = 3, r_3 = 5$, and $r_4 = 4$ are centered on the four vertices $A, B, C$, and $D$ respectively. Two pairs of external tangents are drawn for the circles at $A$ and $C$ and for the circles at $B$ and $D$. These four tangents intersect to form a quadrilateral $WXYZ$ where $\overline{WX}$ and $\overline{YZ}$ lie on the tangents through the circles on $A$ and $C$. If $\overline{WX} + \overline{YZ} = 20$, find the area of quadrilateral $WXYZ$.




10. In acute $\triangle ABC$, let points $D$, $E$, and $F$ be the feet of the altitudes of the triangle from $A$, $B$, and $C$, respectively. The area of $\triangle AEF$ is 1, the area of $\triangle CDE$ is 2, and the area of $\triangle BFD$ is $2 - \sqrt{3}$. What is the area of $\triangle DEF$?