1. Five people each choose an integer between 1 and 3, inclusive. What is the probability that all 3 numbers are chosen by at least one of the five people?

Answer: $\frac{50}{81}$

Solution: We use complementary counting and find how many cases there are where at least one number is not chosen. Call E_n the event where n is not chosen. Then, using the principle of inclusion and exclusion, $\#(E_1 \cup E_2 \cup E_3) = \#(E_1) + \#(E_2) + \#(E_3) - (\#(E_1 \cup E_2) + \#(E_1 \cup E_3) + \#(E_2 \cup E_3)) + \#(E_1 + E_2 + E_3)$. $\#(E_1) = 2^5$, since there are two numbers left over for the 5 people to choose and similarly, $\#(E_1 \cup E_2) = 1^5$, and $\#(E_1 \cup E_2 \cup E_3) = 0$. Thus there are $3 * 2^5 - 3 = 93$ ways that one of the 3 numbers is not selected. Our final answer is then $P = 1 - \frac{93}{3^5} = \boxed{\frac{50}{81}}$.

2. How many ordered quadruples (a, b, c, d) satisfy $a+b+c+d = 4030$ and $a, b, c, d \in \{-2020, -2019, ..., -2011, -201$ a, b, c, and d do not need to be distinct.

Answer: 3564

Solution: Let $a = a_1 + a_2$ where $a_1 \in \{-2020, 2010\}, a_2 \in \{0, 1, 2...10\}$. Write b, c, d similarly. Of a_1, b_1, c_1, d_1, 3 must equal 2010 and 1 must equal -2020. Thus there are 4 ways to assign values to a_1, b_1, c_1, d_1. Then $a_2 + b_2 + c_2 + d_2 = 4030 - (a_1 + b_1 + c_1 + d_1) = 20$ By sticks and stones, if there was no upper constraint on a_2, b_2, c_2, d_2, there would be $\binom{23}{3} = 1771$ ways to assign values. We now subtract cases where one of the values is greater than 10. Note that we can only have one value greater than 10. WLOG let $a_2 >= 11$. Consider $a_3 = a_2 - 11$ we then want the number of integer solutions to $a_3 + b_2 + c_2 + d_2 = 9$ There are then $\binom{12}{3} = 220$ way. Thus there are a total of $1771 - 4 * 220 = 891$ ways to assign values to a_2, b_2, c_2, d_2. Then there are 891*4=$\boxed{3564}$ quadruples.

3. Adam flips a fair coin. He stops flipping when he flips the same face 2021 consecutive times. If $a^b - 1$ is the expected number of flips, where a and b are positive integers and a is prime, find $a + b$.

Answer: 2023

Solution: Let us define $E(n)$ as a state of n consecutive flips such that it defines the expected number of flips to reach 2021 consecutive results given n consecutive results. Given this definition, $E(0) = R$.

$$E(2021) = 0$$
$$E(2020) = \frac{1}{2}(1 + E(2021)) + \frac{1}{2}(1 + E(1)) = (1 + \frac{1}{2}E(1))$$
$$E(2019) = \frac{1}{2}(1 + E(2020)) + \frac{1}{2}(1 + E(1)) = \frac{1}{2}E(2020) + (1 + \frac{1}{2}E(1))$$
$$\vdots$$
$$E(1) = \frac{1}{2}(1 + E(2)) + \frac{1}{2}(1 + E(1)) = \frac{1}{2}E(2) + (1 + \frac{1}{2}E(1))$$
$$E(0) = 1 + E(1)$$

We know that, for $i \in \{1, 2, 3, ..., 2020\}$, $E(i) = \frac{1}{2}E(i+1) + (1 + \frac{1}{2}E(1))$ holds. Using the fact that $E(i+1) = \frac{1}{2}E(i+2) + (1 + \frac{1}{2}E(1))$ holds for $0 \le i \le 2020$, we have the following:

$$E(i+1)-E(i) = (\frac{1}{2}E(i+2)+(1+\frac{1}{2}E(1)))-(\frac{1}{2}E(i+1)+(1+\frac{1}{2}E(1))) = \frac{1}{2}(E(i+2)-E(i+1))$$

The common difference is geometric, so given $E(2020)-E(2021) = (1+\frac{1}{2}E(1))-0 = 1+\frac{1}{2}E(1)$, we solve for $E(1)$:

$$E(1) = E(1)-E(2021) = (E(1)-E(2)) + (E(2)-E(3)) + ... + (E(2020)-E(2021))$$
$$= (1+\frac{1}{2}+\frac{1}{4}+...+(\frac{1}{2})^{2019})(1+\frac{1}{2}E(1)) = (2-(\frac{1}{2})^{2019})(1+\frac{1}{2}E(1))$$

Solving for $E(1)$, we have

$$(\frac{1}{2})^{2020}E(1) = 2-(\frac{1}{2})^{2019} \implies E(1) = 2^{2021}-2.$$

Then, $E(0) = 1+E(1) = 2^{2021}-1$. The answer is $2+2021 = \boxed{2023}$.