Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Find the remainder when x^6 is divided by $x^2 - 3x + 2$.

2. Compute the sum of possible integers such that $x^4 + 6x^3 + 11x^2 + 3x + 16$ is a square number.

3. Suppose $f(x) = \sqrt{x^2 - 102x + 2018}$. Let A and B be the smallest integer values of the function that can be derived from integer inputs. Given $A < B$, find A and B.

4. Let x and y be complex numbers such that $x^2 + y^2 = 31$ and $x^3 + y^3 = 154$. Find the maximum possible real value of $x + y$.

5. The function $y = x^2$ does not include the point $(5, 0)$. Let θ be the absolute value of the smallest angle the curve needs to be rotated around the origin so that it includes $(5, 0)$?. Find $\tan(\theta)$

6. The polynomial $1 - 2x + 4x^2 - 8x^3 + ... + 2^{20}x^{20} - 2^{21}x^{21}$ can be expressed as $c_0 + c_1y + ... + c_{20}y^{20} + c_{21}y^{21}$ where $y = x + \frac{1}{2}$. Find c_2.

7. Let x, y, and z be positive real numbers with $1 < x < y < z$ such that
$$\log_x y + \log_y z + \log_z x = 8, \text{and}$$
$$\log_x z + \log_z y + \log_y x = \frac{25}{2}.$$
The value of $\log_y z$ can then be written as $\frac{p+\sqrt{q}}{r}$ for positive integers p, q, and r such that q is not divisible by the square of any prime. Compute $p + q + r$.

8. Find the sum of all possible values of a such that there exists a non-zero complex number z such that the four roots, labeled r_1 through r_4, of the polynomial
$$x^4 - 6ax^3 + (8a^2 + 5a)x^2 - 12a^2x + 4a^2$$
satisfy $|\Re(r_i)| = |r_i - z|$ for $1 \leq i \leq 4$. Note, for a complex number x, $\Re(x)$ denotes the real component of x.

9. Let $m, n \subset \mathbb{R}$ and
$$f(m, n) = m^4(8 - m^4) + 2m^2n^2(12 - m^2n^2) + n^4(18 - n^4) - 100$$
Find the smallest possible value for a in which $f(m, n) \leq a$, regardless of the input of f.

10. Suppose that the polynomial $x^2 + ax + b$ has the property such that if s is a root, then $s^2 - 6$ is a root. What is the largest possible value of $a + b$?