**Time limit:** 110 minutes.
**Instructions:** This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
**No calculators.**

1. In Trolliland, every troll lives under a custom bridge which is a half circle with radius 1 foot more than the height of the troll. If there are 75 trolls in Trolliland and an average height of 3 feet, what is the total lengths of all of the troll bridges in Trolliland?

2. Leonard is standing at the origin in 3D space. He can only move forward one unit in the x-direction, the y-direction, or the z-direction. How many ways can he get to (3, 3, 3)?

3. How many dates can be formed with only the digits 2 and 0 that are in the future in comparison to today?

4. Consider the sequence $a_n = a_{n-1} + 22a_{n-2} + 2020a_{n-3}$ for $n \geq 3$ where $a_0 = 1$, $a_1 = 1$, $a_2 = 2$. What is the last digit of $a_{2020}$?

5. Let a number be called awesome if it: (i) is 3-digits in base 12, (ii) is 4-digits in base 7, and (iii) does not have a digit that is 0 in base 10. How many awesome numbers (in base 10) are there?

6. A cat chases a mouse on the integer lattice. The mouse starts at $(0, 0)$ and wants to get to the mouse hole at $(5, 5)$. The cat starts at $(-1, -3)$. The mouse can only travel along the grid at 1 unit/sec, whereas the cat can travel on diagonals and at 2 units/sec. How long will have the cat been waiting at the hole for the mouse?

7. Emily writes down 10 consecutive integers and then Vinjai erases one of the them. If the sum of the remaining 9 numbers is 2020, what number did Vinjai erase?

8. Find the smallest real root of
$$14x^4 - 2x^3 + 13x^2 - 3x - 12$$

9. Colleen and Colin in total have 100 skittles. After Halloween, the number of skittles Colleen has is twice the amount that Colin has. Colin and Colleen got identical candy collections from trick-or-treating. How many possible pairs of number of skittles can Colin and Colleen start with?

10. You can buy packets of 5 cookies or packets of 11 cookies. Assuming an infinite amount of money, what is the largest number of cookies that you cannot buy?

11. If you are making a bracelet with 7 indistinguishable purple beads and 2 indistinguishable red beads, how many distinct bracelets can you make? Assume that reflections and rotations are indistinct.

12. A certain 10-sided die has the number 1 one one side, the number 2 on two sides, the number 3 on three sides, and the number 4 on the the remaining 4 sides. Nathan and David each roll this die once. If the die is equally likely to land on any of the 10 sides, what is the probability that the number Nathan rolled is greater than the number David rolled?

13. Three mutually-tangent circles are inscribed by a larger circle of radius 1. Their centers form a equilateral triangle, whose side length can be written as $a + b\sqrt{3}$, where $a$ and $b$ are rational numbers. What is $ab$?

14. How many factors of $20^{20}$ are greater than 2020?

15. Express $\sqrt{\frac{43}{4} + \frac{15}{\sqrt{2}}}$ in the form $\frac{a+b\sqrt{c}}{d}$, where $a, b, c, d$ are integers, $c$ is square-free, $a$ and $d$ are relatively prime, and $b$ and $d$ are relatively prime.

16. Consider triangle $ABC$ on the coordinate plane with $A = (2, 3)$ and $C = (\frac{96}{13}, \frac{207}{13})$. Let $B$ be the point with the smallest possible y-coordinate such that $AB = 13$ and $BC = 15$. Compute the coordinates of the incenter of triangle $ABC$.

17. Let $i = \sqrt{-1}$. Compute the number of ordered pairs of positive integers $(a, b)$ such that the complex number $z = a + bi$ satisfies $|2 + z|^2 + |2 - z|^2 < 50$.

18. Let $x$ and $y$ be positive numbers such that $xy + x + y = 80$ and $x = 13y$. Moreover, suppose $a$ and $b$ are real numbers such that $a + b = x^2$ and $ab = 2019$. Write out all possible solutions $(a, b)$.

19. Suppose $ABCD$ is a square with points $E, F, G, H$ inside square $ABCD$ such that $ABE$, $BCF$, $CDG$, and $DAH$ are all equilateral triangles. Let $E', F', G', H'$ be points outside square $ABCD$ such that $ABE'$, $BCF'$, $CDG'$, and $DAH'$ are also all equilateral triangles. What is the ratio of the area of quadrilateral $EFGH$ to the area of the quadrilateral $E'F'G'H'$?

20. Square $ABCD$ has side length 4. Points $P$ and $Q$ are located on sides $BC$ and $CD$, respectively, such that $BP = DQ = 1$. Let $AQ$ intersect $DP$ at point $X$. Compute the area of triangle $PQX$.

21. Compute the remainder of $2^{10} + 2^{11} + 5^{10} + 5^{11} + 10^{10} + 10^{11}$ when divided by 13.

22. Given that $1A345678B0$ is a multiple of 2020, compute $10A + B$.

23. 3 points are randomly selected from the vertices from a regular 2020-gon. What is the probability the three vertices form a scalene triangle?

24. Suppose the absolute difference between the area and perimeter of a rectangle with integer side lengths is 2020. What is the minimum possible value of the perimeter of this rectangle?

25. Find the number of subsets $S$ of $\{1, 2, \ldots, 10\}$ such that no two of the elements in $S$ are consecutive.