Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Katy writes down an odd composite positive integer less than 1000. Katy then generates a new integer by reversing the digits of her initial number. The new number is a multiple of 25 and is also less than her initial number. What was the initial number that Katy wrote down?

2. Find the probability that a randomly selected divisor of 20! is a multiple of 2000.

3. Let $A(n) = \sum_{i=1}^{n} \lceil \frac{n}{i} \rceil$ and $B(n) = \sum_{i=1}^{n} \lfloor \frac{n}{i} \rfloor$. Compute $A(2020) - B(2020)$.

4. What is the smallest positive multiple of 2020 that has all distinct digits?

5. Find the smallest integer value of n such that

$$\underbrace{2^{2^{\cdots^{2}}}}_{n\ 2\text{'s}} \geq 16^{16^{16^{16}}}.$$

6. William has a bag of white, milk, and dark chocolate bars. Each minute he reaches into the bag, selects a chocolate bar at random, and eats it. Given that there are 17 milk chocolate bars, 12 dark chocolate bars, and 19 white chocolate bars, what is the probability that William runs out of milk chocolate bars first and dark chocolate bars second?

7. A rook is on a chess board with 8 rows and 8 columns. The rows are numbered 1, 2, ..., 8 and the columns are lettered a, b, ..., h. The rook begins at a1 (the square in both row 1 and column a). Every minute, the rook randomly moves to a different square either in the same row or the same column. The rook continues to move until it arrives a square in either row 8 or column h. After infinite time, what is the probability the rook ends at a8?

8. Suppose Joey is at the origin and wants to walk to the point $(20, 20)$. At each lattice point, Joey has three possible options. He can either travel 1 lattice point to the right, 1 lattice point above, or diagonally to the lattice point 1 above and 1 to the right. If none of the lattice points Joey reaches have their coordinates sum to a multiple of 3 (excluding his starting point), how many different paths can Joey take to get to $(20, 20)$?

9. Elena and Mina are making volleyball teams for a tournament, so they find 15 classmates and have them stand in a line from tallest to shortest. They each select six students, such that no two students on the same team stood next to each other in line. How many ways are there to choose teams?

10. Suppose n is a product of three primes p_1, p_2, p_3 where $p_1 < p_2 < p_3$ and p_1 is a two-digit integer. If $n - 1$ is a perfect square, compute the smallest possible value of n.