1. Compute
$$\int_1^2 \frac{x-1}{x^2\ln x + x}dx.$$

Answer: $\mathbf{\ln(\frac{1}{2} + \ln 2)}$

Solution: Notice that we can rewrite this as
$$\int_1^2 \frac{\frac{1}{x} - \frac{1}{x^2}}{\ln x + \frac{1}{x}}dx,$$
from which the u-substitution $u = \ln x + \frac{1}{x}$ shows that the integral is just $\ln(\frac{1}{x} + \ln x)$. We get the answer by evaluating at the appropriate endpoints.

2. Calculate
$$\sum_{n=1}^{\infty} \frac{\sin n + \cos n}{n}.$$

Answer: $\mathbf{\frac{\pi-1}{2} - \frac{1}{2}\ln(2 - 2\cos 1)}$

Solution: Recall that power series tell us that $-\log(1-z) = \sum_{n=1}^{\infty} \frac{z^n}{n}$, and recall that we can write $\sin x$ as $\frac{1}{2i}(e^{ix} - e^{-ix})$ and $\cos x$ as $\frac{1}{2}(e^{ix} + e^{-ix})$. Combining these facts, we see that the infinite sum of the cosine portion is equal to $\frac{1}{2}(-\ln(1-e^i) - \ln(1-e^{-i})) = -\frac{1}{2}\ln((1-e^i)(1-e^{-i})) = -\frac{1}{2}\ln(2-2\cos 1)$. Similarly, we see that the sine portion of the infinite sum is $\frac{1}{2i}(-\ln(1-e^i) + \ln(1-e^{-i})) = \frac{1}{2i}\ln(-e^{-i}) = \frac{\pi-1}{2}$. Combining these two portions gives the desired answer.

3. Let
$$y = \sum_{n=0}^{\infty} \frac{n+x}{n!} \cdot 2^n$$
describe a curve in the xy plane. Find the area under the curve from $x = 0$ to $x = 2020$.

Answer: $\mathbf{2044240e^2}$

Solution: We note that this is the Taylor series evaluated at $z = 2$ of
$$x\sum_{n=0}^{\infty} \frac{z^n}{n} + \sum_{n=1}^{\infty} \frac{z^n}{(n-1)!} = xe^2 + 2e^2 = xe^2 + 2e^2$$
. So, we calculate the area:
$$\int_0^{2020} xe^2 + 2e^2 dx = \frac{2020^2 e^2}{2} + 4040e^2 = (2040200 + 4040)e^2 = \boxed{2044240e^2}$$