

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. Consider a semi-circle with diameter AB . Let points C and D be on diameter AB such that CD forms the base of a square inscribed in the semicircle. Given that $CD = 2$, compute the length of AB .
2. Let $ABCD$ be a trapezoid with AB parallel to CD and perpendicular to BC . Let M be a point on BC such that $\angle AMB = \angle DMC$. If $AB = 3$, $BC = 24$, and $CD = 4$, what is the value of $AM + MD$?
3. Let ABC be a triangle and D be a point such that A and D are on opposite sides of BC . Give that $\angle ACD = 75^\circ$, $AC = 2$, $BD = \sqrt{6}$, and AD is an angle bisector of both $\triangle ABC$ and $\triangle BCD$, find the area of quadrilateral $ABDC$.
4. Let a_1, a_2, \dots, a_{12} be the vertices of a regular dodecagon D_1 (12-gon). The four vertices a_1, a_4, a_7, a_{10} form a square, as do the four vertices a_2, a_5, a_8, a_{11} and a_3, a_6, a_9, a_{12} . Let D_2 be the polygon formed by the intersection of these three squares. If we let $[A]$ denotes the area of polygon A , compute $\frac{[D_2]}{[D_1]}$.
5. In $\triangle ABC$, $\angle ABC = 75^\circ$ and $\angle BAC$ is obtuse. Points D and E are on AC and BC , respectively, such that $\frac{AB}{BC} = \frac{DE}{EC}$ and $\angle DEC = \angle EDC$. Compute $\angle DEC$ in degrees.
6. In $\triangle ABC$, $AB = 3$, $AC = 6$, and D is drawn on BC such that AD is the angle bisector of $\angle BAC$. D is reflected across AB to a point E , and suppose that AC and BE are parallel. Compute CE .
7. Two equilateral triangles ABC and DEF , each with side length 1, are drawn in 2 parallel planes such that when one plane is projected onto the other, the vertices of the triangles form a regular hexagon $AFBDCE$. Line segments AE , AF , BF , BD , CD , and CE are drawn, and suppose that each of these segments also has length 1. Compute the volume of the resulting solid that is formed.
8. Let ABC be a right triangle with $\angle ACB = 90^\circ$, $BC = 16$, and $AC = 12$. Let the angle bisectors of $\angle BAC$ and $\angle ABC$ intersect BC and AC at D and E respectively. Let AD and BE intersect at I , and let the circle centered at I passing through C intersect AB at P and Q such that $AQ < AP$. Compute the area of quadrilateral $DPQE$.
9. Let $ABCD$ be a cyclic quadrilateral with $3AB = 2AD$ and $BC = CD$. The diagonals AC and BD intersect at point X . Let E be a point on AD such that $DE = AB$ and Y be the point of intersection of lines AC and BE . If the area of triangle ABY is 5, then what is the area of quadrilateral $DEYX$?
10. Let ABC be a triangle with $AB = 13$, $AC = 14$, and $BC = 15$, and let Γ be its incircle with incenter I . Let D and E be the points of tangency between Γ and BC and AC respectively, and let ω be the circle inscribed in $CDIE$. If Q is the intersection point between Γ and ω and P is the intersection point between CQ and ω , compute the length of PQ .