1. If a, b, c are real numbers with $a - b = 4$, find the maximum value of $ac + bc - c^2 - ab$.

   Answer: 4

   Solution: We have

$$\begin{aligned} ac + bc - c^2 - ab &= (a-c)(c-b) \\ &= \left(\sqrt{(a-c)(c-b)}\right)^2 \\ &\le \left(\frac{(a-c)+(c-b)}{2}\right)^2 \\ &= \left(\frac{a-b}{2}\right)^2 \\ &= \left(\frac{4}{2}\right)^2 \\ &= \boxed{4} \end{aligned}$$

   The maximum is attained when $a = 4$, $b = 0$, and $c = 2$.

2. If $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$ and $\frac{1}{x+1} + \frac{1}{y+1} = \frac{3}{8}$, compute $\frac{1}{x-1} + \frac{1}{y-1}$.

   Answer: $\frac{11}{14}$

   Solution: With two equations and two unknowns, we can solve for the expressions $a = xy$ and $b = x + y$. The first equation can be written as

$$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{b}{a} = \frac{1}{2}$$

   which implies that $a = 2b$. Similarly, from the second equation we have

$$\frac{1}{x+1} + \frac{1}{y+1} = \frac{(x+y)+2}{xy+(x+y)+1} = \frac{b+2}{a+b+1} = \frac{3}{8}$$

   which implies that $8(b + 2) = 3(a + b + 1)$, or $3a - 5b = 13$. Solving the system of equations gives us $a = 26$ and $b = 13$. Therefore, we have

$$\frac{1}{x-1} + \frac{1}{y-1} = \frac{(x+y)-2}{xy-(x+y)+1} = \frac{b-2}{a-b+1} = \boxed{\frac{11}{14}}.$$

3. Let F_n denote the n-th term of the Fibonacci sequence defined recursively as $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for all $n \ge 0$. Compute the sum

$$\sum_{n=1}^{\infty} \frac{F_n}{2^n}$$

   Answer: 2

Let S be the desired sum. Note that

$$\begin{aligned}
S - \frac{S}{2} &= \sum_{n=1}^{\infty} \frac{F_n}{2^n} - \sum_{n=1}^{\infty} \frac{F_n}{2^{n+1}} \\
&= \sum_{n=1}^{\infty} \frac{F_n}{2^n} - \sum_{n=2}^{\infty} \frac{F_{n-1}}{2^n} \\
&= \frac{F_1}{2} + \frac{F_2}{4} + \sum_{n=3}^{\infty} \frac{F_n}{2^n} - \frac{F_1}{4} - \sum_{n=3}^{\infty} \frac{F_{n-1}}{2^n} \\
&= \frac{1}{2} + \sum_{n=3}^{\infty} \frac{F_{n-2}}{2^n} \\
&= \frac{1}{2} + \frac{1}{4} \sum_{n=1}^{\infty} \frac{F_n}{2^n} \\
&= \frac{1}{2} + \frac{S}{4}
\end{aligned}$$

Therefore, we have $\frac{S}{2} = \frac{1}{2} + \frac{S}{4}$, which solves to $S = \boxed{2}$.