

1. There is a circular table with 6 chairs. Alice, Bob, and Eve each pick a random chair to sit down in. Compute the probability that each person is sitting next to two empty chairs.

Answer: $\frac{1}{10}$

Solution: If Alice picks a seat at random to sit down in, Bob then has probability $\frac{2}{5}$ of sitting down in a valid seat (since if he sits directly across from Alice, Eve will be forced to sit next to one of them), and Eve then has probability $\frac{1}{4}$ of sitting down in a valid seat, for a probability of $\boxed{\frac{1}{10}}$.

2. $ABCDEF$ is a regular hexagon, and diagonal AC has length 6. Find the area of hexagon $ABCDEF$.

Answer: $18\sqrt{3}$

Solution: Let M be the midpoint of AC . Then ABM is a $30 - 60 - 90$ right triangle. Since $AM = 3$, $AB = 2\sqrt{3}$, so the area of an equilateral triangle with side length AB is $(2\sqrt{3})^2 \cdot \frac{\sqrt{3}}{4} = 3\sqrt{3}$. Hexagon $ABCDEF$ is composed of six such triangles, for a total area of $\boxed{18\sqrt{3}}$.

3. Let n be the number which consists of the first 2014 positive integers concatenated together. Let $f(x)$ be the sum of the digits of x , and let $g(x)$ be the value obtained by applying f repeatedly to x until it converges to a single value. Compute $g(n)$.

Answer: 1

Solution: Two integers are equivalent mod some integer k if they have the same remainder upon division by k . Note that $g(x)$ is equivalent to $f(x) \pmod{9}$. Furthermore, $f(x)$ should converge to a single-digit number. Since the sum of the digits $1 - 9$ is $0 \pmod{9}$, the sum of the digits of the numbers $1 - 999$ is also $0 \pmod{9}$ and the sum of the digits of the numbers $1000 - 1999$ is $1000 \equiv 1 \pmod{9}$. Finally, we can calculate that the sum of the digits of the numbers $2000 - 2014$ is $0 \pmod{9}$. Thus, the sum of the digits of n is $1 \pmod{9}$, and since a number mod 9 is equivalent to the sum of its digits mod 9, it follows that $g(x) = \boxed{1}$.