**Time limit:** 50 minutes.
**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
**No calculators.**

1. If $f(x) = (x-1)^4(x-2)^3(x-3)^2$, find $f'''(1) + f''(2) + f'(3)$.

2. A trapezoid is inscribed in a semicircle of radius 2 such that one base of the trapezoid lies along the diameter of the semicircle. Find the largest possible area of the trapezoid.

3. A sector of a circle has angle $\theta$. Find the value of $\theta$, in radians, for which the ratio of the sector's area to the square of its perimeter (the arc along the circle and the two radial edges) is maximized. Express your answer as a number between 0 and $2\pi$.

4. Let $f(x) = \frac{x^3 e^{x^2}}{1-x^2}$. Find $f^{(7)}(0)$, the 7th derivative of $f$ evaluated at 0.

5. The real-valued infinitely differentiable function $f(x)$ is such that $f(0) = 1$, $f'(0) = 2$, and $f''(0) = 3$. Furthermore, $f$ has the property that
$$f^{(n)}(x) + f^{(n+1)}(x) + f^{(n+2)}(x) + f^{(n+3)}(x) = 0$$
for all $n \geq 0$, where $f^{(n)}(x)$ denotes the $n$th derivative of $f$. Find $f(x)$.

6. Compute $\displaystyle\int_{-\pi}^{\pi} \frac{x^2}{1 + \sin x + \sqrt{1 + \sin^2 x}}\, dx$.

7. For the curve $\sin(x) + \sin(y) = 1$ lying in the first quadrant, find the constant $\alpha$ such that
$$\lim_{x \to 0} x^{\alpha} \frac{d^2 y}{dx^2}$$
exists and is nonzero.

8. Compute $\displaystyle\int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x^2 - x + 1}\, dx$.

9. Solve the integral equation
$$f(x) = \int_0^x e^{x-y} f'(y)\, dy - (x^2 - x + 1)e^x.$$

10. Compute the integral
$$\int_0^{\pi} \ln(1 - 2a\cos x + a^2)\, dx$$
for $a > 1$.