1. George is drawing a Christmas tree; he starts with an isosceles triangle AB_0C_0 with $AB_0 = AC_0 = 41$ and $B_0C_0 = 18$. Then, he draws points B_i and C_i on sides AB_0 and AC_0, respectively, such that $B_iB_{i+1} = 1$ and $C_iC_{i+1} = 1$ ($B_{41} = C_{41} = A$). Finally, he uses a green crayon to color in triangles $B_iC_iC_{i+1}$ for i from 0 to 40. What is the total area that he colors in?

   Answer: $\mathbf{\frac{7560}{41}}$

   Solution: Since B_iC_i is parallel to B_0C_0, triangles AB_iC_i are similar to $\triangle AB_0C_0$. So, the area of $\triangle AB_iC_i$ is $(\frac{41-i}{41})^2$ of the area of AB_0C_0. Also, the area of $\triangle B_iC_iC_{i+1}$ is $\frac{1}{41-i}$ of the area of $\triangle AB_iC_i$. So, the area of $\triangle B_iC_iC_{i+1}$ is $\frac{1}{41-i}(\frac{41-i}{41})^2 = \frac{41-i}{41^2}$.

   The sum of all triangles $\triangle B_iC_iC_{i+1}$ is then $\sum_{i=1}^{41} \frac{i}{41^2} = \frac{\frac{41\times 42}{2}}{41^2} = \frac{21}{41}$. The height of $\triangle AB_0C_0$ is $\sqrt{41^2 - 9^2} = 40$, so its area is $\frac{1}{2} \times 40 \times 18 = 360$. The total area of the colored triangles is $\frac{21}{41} \times 360 = \boxed{\frac{7560}{41}}$.

2. The incircle of $\triangle ABC$ is centered at I and is tangent to BC, CA, and AB at D, E, and F, respectively. A circle with radius 2 is centered at each of D, E, and F. Circle D intersects circle I at points D_1 and D_2. The points E_1, E_2, F_1, and F_2 are defined similarly. If the inradius of $\triangle ABC$ is 5, what is the ratio of the area of the triangle whose sides are formed by extending D_1D_2, E_1E_2, and F_1F_2 to the area of $\triangle ABC$?

   Answer: $\mathbf{\frac{529}{625}}$

   Solution: Let the new triangle be $\triangle XYZ$. Note that $\triangle XYZ \sim \triangle ABC$ since all of its sides are parallel to a corresponding side of $\triangle ABC$. The incenter of $\triangle XYZ$ is also I, so it suffices to find the inradius of $\triangle XYZ$ and then use the ratio with the inradius of $\triangle ABC$ to find the ratio of their areas. Consider circle D. Let the intersection of ID and D_1D_2 be M and let the length of MD be x. Then, $D_1M = \sqrt{4 - x^2}$ and the Pythagorean theorem on $\triangle D_1MI$ gives

$$\begin{aligned} &D_1M^2 + IM^2 = D_1I^2 \\ &\Rightarrow 4 - x^2 + (5 - x)^2 = 25 \\ &\Rightarrow 4 - 10x = 0 \\ &\Rightarrow x = \frac{2}{5}. \end{aligned}$$

   Then, $IM = \frac{23}{25}$, which is the inradius of $\triangle XYZ$. The ratio of the areas of $\triangle XYZ$ and $\triangle ABC$ is then $\boxed{\frac{529}{625}}$.

3. Let $\triangle ABC$ be a triangle with $BA < AC$, $BC = 10$, and $BA = 8$. Let H be the orthocenter of $\triangle ABC$. Let F be the point on segment AC such that $BF = 8$. Let T be the point of intersection of FH and the extension of line BC. Suppose that $BT = 8$. Find the area of $\triangle ABC$.

   Answer: $\mathbf{15\sqrt{7}}$

   Solution: We claim that $\triangle TAC$ is isosceles. It will suffice to show that $\triangle THC$ is isosceles. Note that $BHFC$ is cyclic. Hence $\angle BFH = \angle BHC$. But then $\triangle TBF \sim \triangle THC$, from AA similarity. Since $\triangle TBF$ is isosceles, so is $\triangle THC$. Hence we have that AH is the perpendicular bisector of TC which has length 18. Let A_H be the foot of the altitude from A to BC. We then see that $A_HC = 9$, and so $BA_H = 1$. From the Pythaogrean Theorem, we then have that $AA_H = \sqrt{63}$, and so our answer is $\frac{1}{2} \cdot 10 \cdot \sqrt{63} = 15\sqrt{7}$.