Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Compute the minimum possible value of

$$(x-1)^2+(x-2)^2+(x-3)^2+(x-4)^2+(x-5)^2$$

for real values of x.

2. Find all real values of x such that $(\frac{1}{5}(x^2-10x+26))^{x^2-6x+5}=1$.

3. Express $\frac{2^3-1}{2^3+1}\times\frac{3^3-1}{3^3+1}\times\frac{4^3-1}{4^3+1}\times\cdots\times\frac{16^3-1}{16^3+1}$ as a fraction in lowest terms.

4. If x, y, and z are integers satisfying $xyz+4(x+y+z)=2(xy+xz+yz)+7$, list all possibilities for the ordered triple (x, y, z).

5. The quartic (4th-degree) polynomial $P(x)$ satisfies $P(1)=0$ and attains its maximum value of 3 at both $x=2$ and $x=3$. Compute $P(5)$.

6. There exist two triples of real numbers (a, b, c) such that $a-\frac{1}{b}$, $b-\frac{1}{c}$, and $c-\frac{1}{a}$ are the roots to the cubic equation $x^3-5x^2-15x+3$ listed in increasing order. Denote those (a_1, b_1, c_1) and (a_2, b_2, c_2). If a_1, b_1, and c_1 are the roots to monic cubic polynomial f and a_2, b_2, and c_2 are the roots to monic cubic polynomial g, find $f(0)^3+g(0)^3$.

7. The function $f(x)$ is known to be of the form $\prod_{i=1}^{n} f_i(a_i x)$, where a_i is a real number and $f_i(x)$ is either $\sin(x)$ or $\cos(x)$ for $i=1,\ldots,n$. Additionally, $f(x)$ is known to have zeros at every integer between 1 and 2012 (inclusive) except for one integer b. Find the sum of all possible values of b.

8. For real numbers (x, y, z) satisfying the following equations, find all possible values of $x+y+z$.

$$\begin{aligned} x^2y+y^2z+z^2x &= -1 \\ xy^2+yz^2+zx^2 &= 5 \\ xyz &= -2 \end{aligned}$$

9. Find the minimum value of xy, given that $x^2+y^2+z^2=7$, $xy+xz+yz=4$, and x, y, z are real numbers.

10. Let $X_1, X_2, \ldots, X_{2012}$ be chosen independently and uniformly at random from the interval $(0, 1]$. In other words, for each X_n, the probability that it is in the interval $(a, b]$ is $b-a$. Compute the probability that $\lceil\log_2 X_1\rceil+\lceil\log_4 X_2\rceil+\cdots+\lceil\log_{4024} X_{2012}\rceil$ is even. (Note: For any real number a, $\lceil a\rceil$ is defined as the smallest integer not less than a.)